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Numerical modelling of overtaking collisions of dust acoustic waves in plasmas

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Abstract. The overtaking collision between two single and unidirectional dust acoustic waves in dusty plasmas consisting of Boltzmann electrons and ions, and negative dust grains has been investigated by PIC simulation method. The well-known physical phenomenon is that the larger soliton moves faster, approaches the smaller one and after the overtaking collision both resume their original shape and speed with different phase shifts. The merging amplitude of two solitons and phase shifts of solitons after collision are given. These PIC results are compared with the overtaking collision of two-soliton solution (TSS) of KdV equaiton obtained by Hirota bilinear method. Comparisons between two indicates that if the amplitude of fast soliton is large enough or the amplitude of slow soliton is small enough, the simulation results are consistent with the interaction of Hirota results.

1 Introduction

The study of nonlinear structures in various kinds of plasmas has become one of the most important topics in the plasma physics due to their relevance in microwave transmission [1–4], confinement fusion [5–9], and astrophysical and space environments [10–13] such as asteroid zones, comettails, interstellar medium, magnetosphere radio frequency plasma discharge, planetary ring and lower part of Earth's ionosphere. In dusty plasmas, because of the presence of a high density of dust grains, there exist different types of collective processes and new wave modes can be exciting. One of these modes is the low-frequency dust acoustic waves (DAWs), first time theoretically predicted by Rao et al. [14], and then experiments were conducted to investigate the DA waves [15]. During the past few years, the many properties of DA modes have been extensively studied [16–22].

Different plasma models appear owing to a delicate balance between dispersion and nonlinearity, which results in the generation of solitons. At present, interaction in process of soliton propagation stands for one of the most important and interesting nonlinear phenomenon in modern plasma researches. Zabusky and Kruskal [23] were first to discuss that when solitons usually defined as a type of solitary waves undergo a collision then they preserve their shape and velocities after the collision. For the collision of solitons, some phenomena have been observed in the laboratory [24–26] and the same can be explained in the multiple solutions of nonlinear evolution equations which can be obtained by various powerful transformation methods like the Hirota bilinear method [27], the inverse scattering method [28], the Darboux transformation method [29,30], the Bäcklund transformation method [31,32], etc. In a one-(or quasi-one-) dimensional system, the solitons may interact between them in two different ways. One is the head-on collision of multi-solitons traveling in the opposite directions, which has been widely investigated theoretically [33–38] and verified by experiments [39–41] and PIC simulation method [42,43], and the other is the overtaking collision of multi-solitons traveling in the same directions, which has been mainly studied theoretically [44–47] and was simply investigated by PIC method [48,49].

However, the overtaking collision properties of dust acoustic solitary waves have not been further verified by either experiment or the numerical simulation until now. In this paper, we will study this question by two methods. One is the Hirota bilinear method, the other is the PIC numerical simulation. The Hirota method is the result of the two-soliton solution of the KdV eqaution which is obtained by the reductive perturbation method from the fluid dynamical equations. Therefore, it is the approximated result. The other one is the numerical experiment performed by using the one-dimensional PIC simulation method to study interaction between two solitary waves propagating in the same direction. In simulation, it is noted that two solitons with different amplitudes collide when the largest one of them catches up with the smaller. The large (fast) soliton has a positive phase shift and the small (slow) soliton has a negative phase shift. The amplitude of two single dust acoustic solitons merging and becoming one soliton during the strongest interaction and the phase shifts of KdV solitons during a collision are obtained. These results are compared with the overtaking collision of two-soliton

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solution of KdV equation obtained by the Hirota bilinear method. It is obvious that when the amplitude of fast soliton is large enough or that of slow soliton is small enough, the PIC simulation results are in good agreement with the Hirota's results.

The organization is shown as follows. In Section 2, the KdV equation obtained by reductive perturbation method and the two-soliton solution of KdV solitary wave by Hirota bilinear method are given. In Section 3 the PIC simulation method is presented. In Section 4, the PIC simulation results are given. In additional, the comparisons between Hirota's results and PIC calculation ones are shown. In Section 5, the conclusions are given.

2 Hirota bilinear method

In order to study the overtaking collision of dust acoustic solitary waves, we focus on the cold dust fluid model. The one-dimensional dimensionless equations of the motion of the system are: $\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0$, $\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{\partial \phi}{\partial x}$, and $\frac{\partial^2 \phi}{\partial x^2} = n_d + \nu e^{\beta s \phi} - \mu e^{-s\phi}$ [50,51], where μ and ν are the normalized ion and electron number densities, respectively. $\beta = \frac{T_i}{T_e}$ is the ratio of the ion and electron temperatures, $s = \frac{1}{\mu + \nu\beta}$. Moreover, the following dimensionless variables are adopted: $r \to r/\lambda_d$, $v \to v/C_d$, $t \to t/\omega^{-1}$, $\phi \to \phi e/k_b T_{eff}$ and $n \to n/n_0$, where λ_d, C_d, ω and T_{eff} are Debye length, sound speed, frequency, and effective temperature of dusty plasma, respectively.

According to reductive perturbation method (RPT), we introduce the following stretched coordinates: $\xi = \varepsilon(lx - ct)$ and $\tau = \varepsilon^3 t$. The dependent variables are expanded around the equilibrium values in power of ε: $f = f_0 + \varepsilon^2 f_1 + \varepsilon^4 f_2 + \ldots$ In first order of ε, we obtain $n_{d1} = u_{d1} = -\phi_1$. To the next higher order, we can obtain the KdV equation:

$$\frac{\partial \phi_1}{\partial \tau} + a\phi_1 \frac{\partial \phi_1}{\partial \xi} + b\frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \tag{1}$$

where

$$a = -\frac{l}{2}\left[3 + s^2\left(\beta^2 \nu - \mu\right)\right], b = \frac{l^3}{2}. \tag{2}$$

The result of the Hirota method is obtained by the following process: first, rewrite the KdV equation as:

$$\frac{\partial \phi'}{\partial t'} - 6\phi' \frac{\partial \phi'}{\partial x'} + \frac{\partial^3 \phi'}{\partial x'^3} = 0, \tag{3}$$

by using the transformations of

$$\phi_1 = \frac{1}{b}\left(\frac{6b^2}{a}\right)^{3/5} \phi', \quad \xi = \left(\frac{6b^2}{a}\right)^{1/5} x' \tag{4}$$

and

$$\tau = \frac{1}{b}\left(\frac{6b^2}{a}\right)^{3/5} t'. \tag{5}$$

Then the two-soliton solution of a KdV equation is as follows in the experiment coordinate [52–55]:

$$\phi_H = -8b\left(\frac{6b^2}{a}\right)^{-3/5} \varepsilon^2 \times \frac{g_1^2 f_1 + g_2^2 f_2 + 2(g_2 - g_1)^2 f_1 f_2 + h\left(g_2^2 f_1^2 f_2 + g_1^2 f_2^2 f_1\right)}{\left(1 + f_2 + f_1 + h f_1 f_2\right)^2} \tag{6}$$

where

$$f_i = e^{2g_i\left(4g_i^2 b\left(\frac{6b^2}{a}\right)^{-3/5}\varepsilon^3 t - \left(\frac{6b^2}{a}\right)^{-1/5}\varepsilon(x-t) + s_i\right)} \quad (i = 1, 2), \tag{7}$$

and

$$h = \left(\frac{g_1 - g_2}{g_1 + g_2}\right)^2. \tag{8}$$

If the initial positions of the two solitons satisfy $s_1 < s_2$, we need $g_1 > g_2$ for the two solitons to collide and g_1 and g_2 stand for amplitudes of two solitons. The phase shifts for two solitons are

$$\triangle x_i = \pm\left(\frac{6b^2}{a}\right)^{1/5} \frac{1}{\varepsilon}\frac{1}{g_i} ln \frac{g_1 + g_2}{g_1 - g_2} \tag{9}$$

where positive sign is for large soliton and negative sign for small soliton.

3 Particle-in-cell method

The one-soliton solution of KdV equation in experimental coordinates is

$$\phi_0 = \phi_m sech^2\left[\frac{x - \left(1 + \frac{u_0 \varepsilon^2}{l}\right)t + \delta_0}{D}\right], \tag{10}$$

where

$$\phi_m = 3\frac{u_0 \varepsilon^2}{a}, \quad D = \frac{2}{l}\sqrt{\frac{b}{u_0 \varepsilon^2}}. \tag{11}$$

δ_0 is a constant representing the initial position of solitary wave.

We assume that there are two copropagating KdV solitons labeled by 1 and 2, respectively. Initially, two solitons propagate in the same direction, but with different speeds. Later, the soliton with the higher speed overtakes the one with slower speed. After the overtake process is over, there are phase shifts for both solitons. In this paper, we will use the PIC code [56] to simulate the Poisson-Boltzmann equation by Particle Algorithm. The dusty particles are treated as so-called "super-particles"(SPs) that are kinetic particles, while the electrons and ions are treated as Boltzmann distributed background. Each SP has a weight factor S that specifies the number of real particles it represents. During the PIC simulation, the simulation region is divided into several grid cells where the field quantities like density, potential, electric field are calculated at the grid points and the particle quantities like velocity, density are calculated at the particle locations. Initially, the SPs are distributed in phase space according to a chosen density distribution and velocity distribution to calculate the charge density. Once is obtained, the Poisson-Boltzmann equation will be solved numerically to derive the value of $E(x)$ at grid. If we identify each SP with a label p, and its mass with m_p, velocity with v_p, charge with q_p and position with x_p, the field imposed on each SP can be worked out and each SP will be driven by electric field according to Newton's motion equations:

$$\frac{dx_p}{dt} = v_p \tag{12}$$

$$m_p \frac{dv_p}{dt} = q_p E(x). \tag{13}$$

These equations will be solved numerically via the leap-frog algorithm. At last, the new position and velocity of each SP are obtained, the procedure comes to repeat until the simulation Completes.

The initial conditions are given as follows:

$$\phi = \phi_{01} + \phi_{02}, \tag{14}$$

where

$$\phi_{01} = \frac{3u_{01}\varepsilon_1^2}{a} sech^2 \frac{x - \frac{1+\varepsilon_1^2 u_{01}}{l_1} t + \delta_{01}}{2\sqrt{b/\left(\varepsilon_1^2 l_1^2 u_{01}\right)}} \tag{15}$$

$$\phi_{02} = \frac{3u_{02}\varepsilon_2^2}{a} sech^2 \frac{x - \frac{1+\varepsilon_2^2 u_{02}}{l_2} t + \delta_{02}}{2\sqrt{b/\left(\varepsilon_2^2 l_2^2 u_{02}\right)}} \tag{16}$$

$v = -\phi_{01} - \phi_{02}$ and $n = 1 - \phi_{01} - \phi_{02}$. The chosen parameters are: grid size $dx = 0.02\lambda_d$, time step $dt = dx/1100$, the number of grid cell is $NX = 110\,000$ and the number of SPs in each grid is 50.

We will compare the results between the PIC method and the Hirota method. For simplicity, we take $l_1 = l_2 = 1$ in equations (15) and (16). Suitable values of δ_{01} and δ_{02} are chosen in order that two solitary waves are far apart initially.

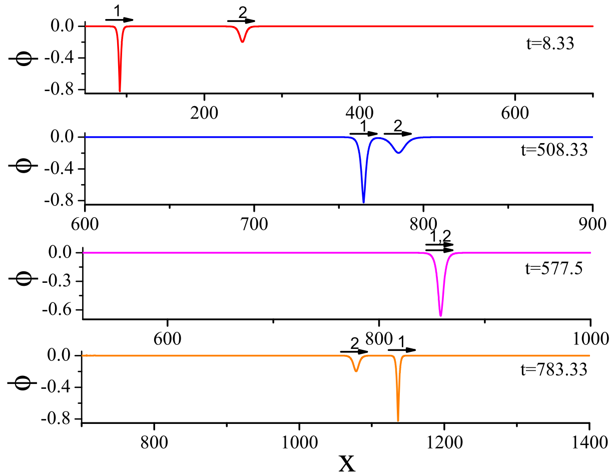

Fig. 1. The waveforms of two colliding solitary waves, 1 and 2, at different times in PIC simulation. $\beta = 0.1$, $\mu = 1.1$ and $\nu = 0.1$.

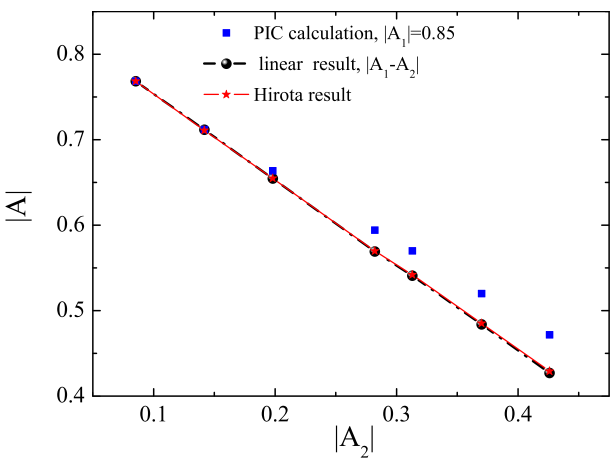

Fig. 2. The dependence of the amplitude of the merged peak in the colliding process on the amplitude of soliton 2 where the amplitude of soliton 1 is fixed. The blue dots are PIC simulation results, the dark yellow ones for linear results $|A_1 - A_2|$ and the red ones for the Hirota results.

4 Pic simulation results

Figure 1 shows the numerical results of the evolution of two dust acoustic solitary waves in different times. Initial amplitudes of colliding solitary waves are about $|A_1| = 0.823$ and $|A_2| = 0.198$, respectively. At the time $t = 577.5$ they collide, merge together and eventually form a symmetric waveform with the amplitude $|A| = 0.66378$. It is found that the merging amplitude is $|A_1 - A_2|$ when the amplitude of soliton 2 is small enough compared with that of soliton 1, for example $|A_2| < 0.14$, where $A_1 = \varepsilon_1^2 \phi_{1m}$ and $A_2 = \varepsilon_2^2 \phi_{2m}$, as shown in Figure 2. Moreover, the dependence of the merging amplitude of two solitons on both the initial amplitudes of two colliding solitons is shown in Figure 3. It is noted that the amplitude of the merged peak in colliding process increases with the amplitude of soliton 1, but decreases with the amplitude of soliton 2.

It also seems from Figure 1 that the larger amplitude soliton moves with the larger speed. It will overtake the smaller amplitude soliton, then both solitons approximately remain their original shape and speed after the overtaking collision is over. However, their trajectories acquire phase shifts. The definitions of phase shifts of both solitary waves are shown in Figure 4. It is obviously observed that the fast soliton has a positive phase shift and the slow soliton has a negative phase shift. It is also noted from Figure 4 that two solitons move faster in PIC simulation than in Hirota method. However, if there is no collision, the soliton 1 moves slower and soliton 2 moves faster.

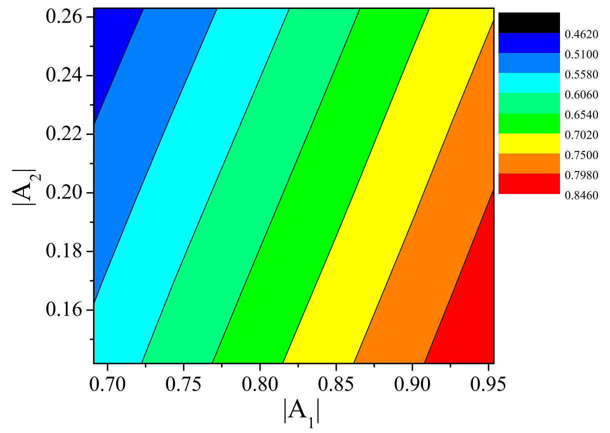

Fig. 3. The amplitude of the merged peak in the colliding process as a function of both amplitudes.

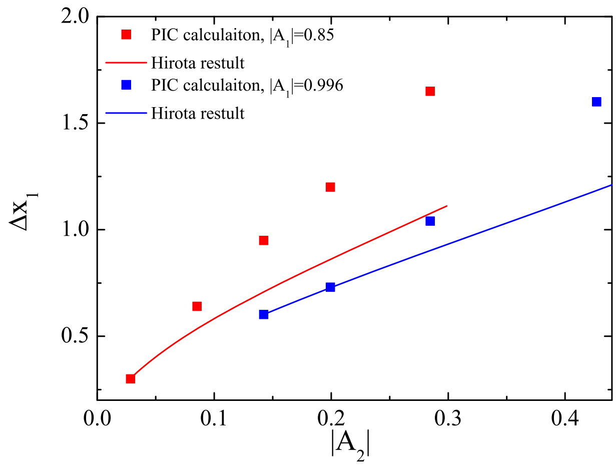

Fig. 5. The phase shift of soliton 1 as a function of the amplitude of soliton 2 for different amplitudes of soliton 1.

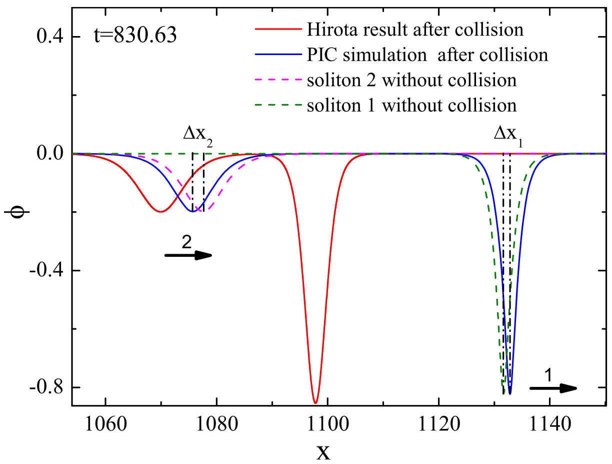

Fig. 4. The definition of the phase shifts of both solitary waves. (a) the blue one for PIC simulation after collision, the Olive one for soliton 1 at the same time but without collision in PIC and the Magenta one for soliton 2 at the same time but without collision in PIC. (b) The red line is the waveform of Hirota result after collision at the same time.

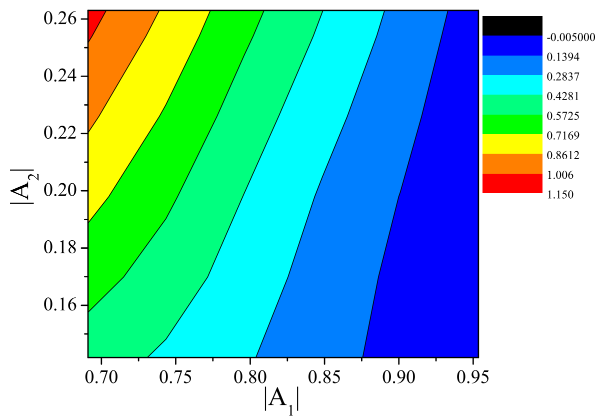

Fig. 6. The differences of the phase shifts of soliton 1 between the numerical results and the Hirota ones as a function of both the amplitudes.

The comparison of phase shifts between the Hirota results and numerical ones (both with the same initial conditions) is shown in Figure 5, which gives the dependence of phase shift Δx_1 of soliton 1 on the amplitude $|A_2|$ of soliton 2 for different amplitudes $|A_1|$ of soliton 1. It is noted that both are in good agreement when the amplitude of soliton 2 is small enough compared with soliton 1. The differences of the phase shifts of soliton 1 between the numerical results and the Hirota ones as a function of the amplitudes of two solitons are shown in Figure 6. It is clear that when the amplitude of soliton 1 is large enough compared with soliton 2, the differences can be neglected. That is to say, when the amplitude of soliton 1 is large enough or the amplitude of soliton 2 is small enough, numerical and Hirota results are consistent. In addition, it is found from Figure 5 that the phase shift of soliton 1 is not only related with its amplitude but also the amplitude of soliton 2, which is consistent with the Hirota results of equation (9). Then the dependencies of phase shift of soliton 1 on both amplitudes are given in Figure 7. It is obviously found that the phase shift of soliton 1 increases with the increasing amplitude of soliton 2, but it decreases with the increase of itself amplitude.

5 conclusions

In this paper, the overtaking collision between two single and unidirectional dust acoustic waves in dusty plasmas consisting of Boltzmann electrons and ions, and negative dust grains has been investigated by PIC simulation method. The well-known physical phenomenon is that the larger soliton moves faster, approaches the smaller one and after the overtaking collision both resume their original shape and speed with different phase shifts. It is found that the phase of the larger soliton will be pushed forward and the smaller one will have a phase lag. The merging

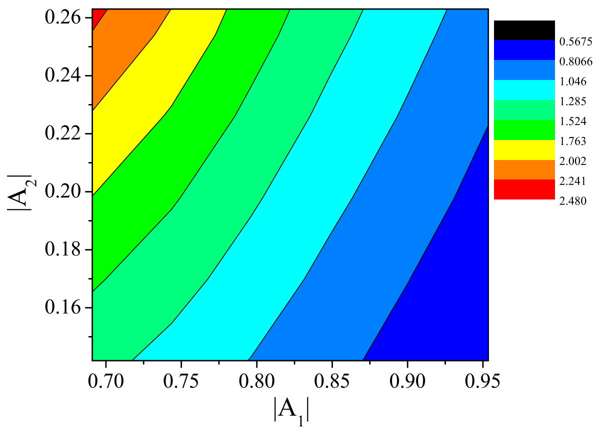

Fig. 7. The dependence of phase shifts of soliton 1 on both amplitudes.

amplitude of two solitons and phase shifts of solitons are given. It is found that the amplitude of the merged peak and phase shift are all related with the amplitudes of two solitons. These PIC results are compared with the overtaking collision of two-soliton solution of KdV equation obtained by Hirota bilinear method. Comparisons between two indicates that if fast soliton is large enough or slow soliton is small enough, the simulation results are in good agreement with the Hirota results. Meanwhile, PIC simulation provides a more realistic description of the dynamics of nonlinear dust acoustic waves that can be usefully applied to various low frequency phenomena observed in laboratory as well as space plasmas. The results have potential applications in the instability study of the space plasmas, and the fusion plasmas. Maybe the results can been realized and tested experimentally according to references [41] and [57].

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