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The Maximum Amplitude of Dust-Acoustic Waves in Dusty Plasma with Vortex-Like Ions

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Abstract

Theoretical investigations are carried out for the properties of small amplitude dust-acoustic solitary waves in plasmas consisting of extremely massive, high negatively charged inertial dust grains, Boltzmann distributed electrons and trapped ions, for one-dimensional case and three-dimensional case. An energy integral equation involving the Sagdeev potential is derived. The dependence of the critical Mach number corresponding to maximum amplitude on other parameters is obtained. It is observed that the magnitude of the external magnetic field has no effect on the amplitude of the solitary waves.

Keywords Complex plasmas · Solitons · Plasma kinetic equations

1 Introduction

Dusty plasma, as mixtures of ordinary plasma particles and charged dust grains, is thought to play an important role in laboratory, astrophysical, and space environments, such as planetary rings, cometary tails, the earth's environment, and interstellar medium [1–6]. Another important feature of dusty plasma is its openness, as it cannot survive without external forces and fluxes [7]. Dusty plasma supports two kinds of acoustic modes [8]. The most well studied of such modes is the so-called dust-acoustic wave (DAW) [9–11] which was discovered since the inception of collective dust-plasma interactions. This is a very low-frequency acoustic mode in which the dust grains participate directly in the wave dynamics [12–19].

Stationary vortex-like distribution is a solitary BGK [20, 21] solution. Define trapping parameter λ such that $|\lambda| = T_{if}/T_{it}$, where T_{if} and T_{it} are the constant temperatures of the free particles and trapped particles, respectively. When trapping parameter $\lambda < 0$ vortex-like distribution exists [22, 23]. A vortex-like distribution of the relevant species can give rise to a depression of the density [24].

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The properties of DAW have been extensively studied in dusty plasmas, especially with trapped particles. Mamun et al. investigated the nonlinear propagation of dust-acoustic waves in a magnetized dusty plasma with vortex-like ion distribution [25] and the nonlinear propagation of dust-acoustic waves in a strongly coupled liquid state dusty plasma with a vortex-like ion distribution [26]. Nonlinear propagation of dust-acoustic waves in an unmagnetized dusty plasma with nonthermal electron and vortex-like ion distribution has been showed by Paul et al. [27]. The instability of DAWs in magnetized vortex-like ion distribution dusty plasmas has been studied [28]. The instability of DAWs in weakly two-dimensional dust plasma with vortex-like ion distribution [29] and the effect of dust size distribution on the propagation of the DAWs in vortex-like ion distribution dusty plasma have also been showed [30]. DAWs in a dusty plasma with charge fluctuation and dust size distribution and vortex-like ion distribution have been presented by Roy Chowdhury [31]. The effects of dust temperature and trapped ions have been incorporated in the study of DAWs by Alinejad [32]. The problem of nonlinear variable charge dust-acoustic waves in a dusty plasma with trapped ions has been revisited by Younsi and Tribeche [33]. The nonlinear properties of DAWs in a magnetized dusty plasma with two-temperature trapped ions have been investigated by Bagchi et al. [34]. Nonlinear solitary oscillations in a varying charge dusty plasma in the presence of nonisothermal trapped electrons have been investigated by Tribeche et al. [35]. Effect of deviations from isothermality of ions on arbitrary amplitude dust-acoustic solitary structures has been studied in an unmagnetized dusty plasma which consists of extremely massive, micron-sized, negatively charged inertial dust grains, Boltzmann distributed electrons and ions with trapped particles, by Alinejad [36], and it is found that the basic properties (Mach number, amplitude, and width) of the solitary structures change drastically due to the effect of deviations from isothermality of ions, and the Mach number increases with the maximum amplitude of solitary wave.

However, Alinejad only confined himself to the one-dimensional case and studied the dependence of the Mach number on the maximum amplitude of arbitrary solitary waves. In this paper, both the one-dimensional case and three-dimensional case for small amplitude waves will be investigated in dusty plasma consisting of extremely massive, micron-sized, negatively charged inertial dust grains, Boltzmann distibuted electrons and vortex-like ions. The Sagdeev potential approach [37–43], which works for arbitrary amplitude solitary waves, has been employed to study the relation of the critical Mach number corresponding to the maximum amplitude of solitary waves with physical quantities. It is observed that the magnitude of the external magnetic field has no effect on the amplitude of the solitary waves.

This paper is organized as follows: In Sections 2 and 3, the basic equations are given. The discussion are given in Section 4. Finally, the conclusion is provided in Section 5.

2 Basic Equations for One-dimensional Case

We consider a three-component plasma consisting of extremely massive, high negatively charged inertial dust grains, Boltzmann distributed electrons and ions trapped particles. Charge neutrality at equilibrium requires that $n_{i0} = Z_d n_{d0} + n_{e0}$, where n_{i0}, n_{e0} and n_{d0} are the unperturbed ion, electron and dust number densities respectively, and Z_d is the number of charges residing on the surface of dust grains.

For one-dimensional case, the dynamics of the nonlinear dust-acoustic waves (with phase lying between the ion and dust thermal velocities) is governed by

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0 \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_d + \nu e^{\beta\phi} - \mu n_i \tag{3}$$

where n_d is the dust particle number density normalized to n_{d0}, n_i is the ion number density normalized to n_{i0}, u_d is the dust fluid velocity normalized to the dust-acoustic speed $C_d = (Z_d T_{if}/m_d)^{1/2}$, and ϕ is the electrostatic wave potential normalized to T_{if}/e, where T_{if} is the constant temperature of the free ions, m_d is the mass of negatively charge dust particulates and e is the magnitude of the electron charge. $\beta = T_{if}/T_e$, with T_e being the electron temperature, $\nu = s/(1-s)$ and $\mu = 1/(1-s)$, where $s = \frac{n_{e0}}{n_{i0}}$. The time and space variables are given in the units of the dust plasma period $\omega_{pd}^{-1} = (m_d/4\pi Z_d^2 n_{d0} e^2)^{1/2}$ and the Debye length $\lambda_{Dd} = (T_{if}/4\pi Z_d n_{d0} e^2)^{1/2}$, respectively. Here the electron inertia is neglected and Boltzmann distribution for the electron density is assumed, which implies isothermality. This expression is obtained by the consideration that a thermal electron moves with a speed much higher than the ion thermal speed. Thus, it would not be much affected by the low-frequency dust-acoustic waves. In this case, ions can interact with the wave potential during its evolution, and therefore can be trapped in the wave potential, leading to a departure from the Boltzmann distribution functions.

To model an ion distribution with deviations from isothermality of ions, we employ a vortex-like distribution of Schamel [20, 44, 45], which solves the ion Vlasov equation. Thus we have $f_i = f_{if} + f_{it}$, where

$$f_{if} = \frac{1}{\sqrt{2\pi}} e^{-(1/2)(v^2+2\phi)} |v| > \sqrt{-2\phi} \tag{4}$$

$$f_{it} = \frac{1}{\sqrt{2\pi}} e^{-\lambda(1/2)(v^2+2\phi)} |v| \leq \sqrt{-2\phi} \tag{5}$$

where f_{if} and f_{it} represents the free and trapped ion contribution, respectively. We note that the distribution function as prescribed above is continuous in the velocity space. Furthermore, the velocity v is normalized to the ion thermal velocity v_{it}. λ stands for the ratio of free ion temperature T_{if} to the trapped ion temperature T_{it}, which determine the number of trapped ions. It is obvious from Eqs. (4) and (5) that $\lambda = 1$ ($\lambda = 0$) represents a Maxwellian (flat-topped) ion distribution, while $\lambda < 0$ represents a vortex-like excavated trapped ion distribution corresponding to an underpopulation of trapped ions [36]. Integrating these ion distributions over the velocity space, we readily obtain the ion number density in the small amplitude limit [36]

$$n_i = 1 - \phi - \frac{4a}{3}(-\phi)^{3/2} + \frac{1}{2}\phi^2 \tag{6}$$

where $a = (1-\lambda)/\sqrt{\pi}$ measures the finite deviation from isothermality of ions. The term $a > 0$, is the contribution of the resonant ions to the ion density.

In order to investigate the properties of small amplitude dust-acoustic solitary waves, we assume that all the dependent variables in Eqs. (1–3) depend only on a single variable $\xi = x - Mt$. We then obtain from Eqs. (1) and (2) the following equation

$$n_d = \frac{1}{\sqrt{1+\frac{2\phi}{M^2}}} \tag{7}$$

$$u_d = M - \sqrt{M^2+2\phi} \tag{8}$$

where we have imposed the appropriate boundary conditions for localized disturbances, i.e, $\phi \to 0$, $n_d \to 1$ and $u_d \to 0$ as $\xi \to \pm\infty$.

Substituting the expressions for n_i, n_e and n_d into Poisson equation of Eq. (3), and integrating it by imposing the boundary conditions for localized solutions, namely, $\phi \to 0$ and $d\phi/d\xi \to 0$ as $\xi \to \pm\infty$, we get a form of "energy conservation":

$$\frac{1}{2}(\frac{d\phi}{d\xi})^2 + V(\phi) = 0 \tag{9}$$

where $V(\phi)$ is the Sagdeev potential

$$V(\phi) = M^2(1-\sqrt{1+\frac{2\phi}{M^2}}) + \frac{\nu}{\beta}(1-e^{\beta\phi}) \tag{10}$$

$$+\mu[\phi - \frac{1}{2}\phi^2 + \frac{8}{15}a(-\phi)^{5/2} + \frac{1}{6}\phi^3]$$

Now, we consider again our general Eq. (9), which can be regarded as an "energy integral" of an oscillating particle of unit mass with a velocity $d\phi/d\xi$ and position ϕ in a potential $V(\phi)$. The features of the soliton can be inferred from the structure of $V(\phi)$. It is obvious from Eq. (10) that $V(\phi) = 0$ and $dV(\phi)/d\phi = 0$ at $\phi = 0$. The existence of a soliton must satisfy the following conditions [46]: (i) $(d^2V/d^2\phi)_{\phi=0} < 0$, so that the fixed point at the origin is unstable; (ii) there exists a nonzero ϕ_m, the maximum (or minimum) value of ϕ, to make $V(\phi_m) = 0$; and (iii)$V(\phi) < 0$ when ϕ lies between 0 and ϕ_m.

As shown in Refs. [40] and [41], there is a critical Mach number M_c corresponding to $\phi_m = -M_c^2/2$, where the potential $V(\phi_m) = 0$. Then we obtain the following equation:

$$M_c^2 + \frac{1}{\beta}\frac{s}{1-s}(1-e^{-\frac{1}{2}\beta M_c^2}) + \frac{1}{1-s}[\frac{-M_c^2}{2} \tag{11}$$

$$-\frac{M_c^4}{8} + \frac{8}{15}\frac{1-\lambda}{\sqrt{\pi}}(\frac{M_c^2}{2})^{5/2} + \frac{1}{6}(-\frac{M_c^2}{2})^3] = 0$$

3 Basic Equations for Three-dimensional Case

For three-dimensional low-frequency acoustic motions, for simplicity, we assume that the acoustic wave is taken unidirectional propagating along z-direction (hence setting $\partial/\partial x = 0$, $\partial/\partial y = 0$ [47–49]) and the external static magnetic field is directed along the z axis, i.e., $\vec{B} = B_0\vec{k}$ (where B_0 is the strength of the magnetic field and $\vec{k}$ is a unit vector along the z-direction). Then we have the following normalized equations for the cold dust fluid [25]:

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d w_d)}{\partial z} = 0 \tag{12}$$

$$\frac{\partial u_d}{\partial t} + w_d\frac{\partial u_d}{\partial z} = -\omega_{cd} v_d \tag{13}$$

$$\frac{\partial v_d}{\partial t} + w_d\frac{\partial v_d}{\partial z} = \omega_{cd} u_d \tag{14}$$

$$\frac{\partial w_d}{\partial t} + w_d\frac{\partial w_d}{\partial z} = \frac{\partial \phi}{\partial z} \tag{15}$$

$$\frac{\partial^2\phi}{\partial z^2} = n_d + \nu e^{\beta\phi} - \mu n_i \tag{16}$$

where $\vec{u_d} = u_d\vec{i} + v_d\vec{j} + w_d\vec{k}$. $\omega_{cd} = (\frac{Z_d}{m_d}eB_0)/\omega_{pd}$ is the dust cyclotron frequency normalized to ω_{pd}. Other quantities are all normalized, which is identical to the case of one dimension, discussed above.

We assume that all the dependent variables in Eqs. (12–16) depend only on a single variable $\xi = z - Mt$, where ξ is normalized by λ_{Dd} and M is the Mach number (solitary wave velocity/ C_d). Then, using the steady state condition and imposing the appropriate boundary conditions for localized perturbations, namely, $\phi \to 0$ and $d\phi/d\xi \to 0$ as $\xi \to \pm\infty$, we get a form of "energy conservation" for arbitrary amplitudes:

$$\frac{1}{2}(\frac{d\phi}{d\xi})^2 + V(\phi) = 0 \tag{17}$$

where $V(\phi)$ is the Sagdeev potential

$$V(\phi) = M^2(1-\sqrt{1+\frac{2\phi}{M^2}}) + \frac{\nu}{\beta}(1-e^{\beta\phi}) \tag{18}$$

$$+\mu[\phi - \frac{1}{2}\phi^2 + \frac{8}{15}a(-\phi)^{5/2} + \frac{1}{6}\phi^3]$$

which is identical to Eq. (10).

By using Eqs. (13) and (14), we obtain

$$u_d = v_d = 0 \tag{19}$$

4 Discussion

For the one-dimensional case, it is noted from Eq. (11) that the critical Mach number M_c corresponding to maximum amplitude for one-dimensional case depends on the ratio of the free ion temperature to the electron temperature $\beta = T_{if}/T_e$, the ratio of the unperturbed electron number density to the ion number density $s = \frac{n_{e0}}{n_{i0}}$ and the ratio of the free ion temperature to the trapped ion temperature $\lambda = \frac{T_{if}}{T_{it}}$.

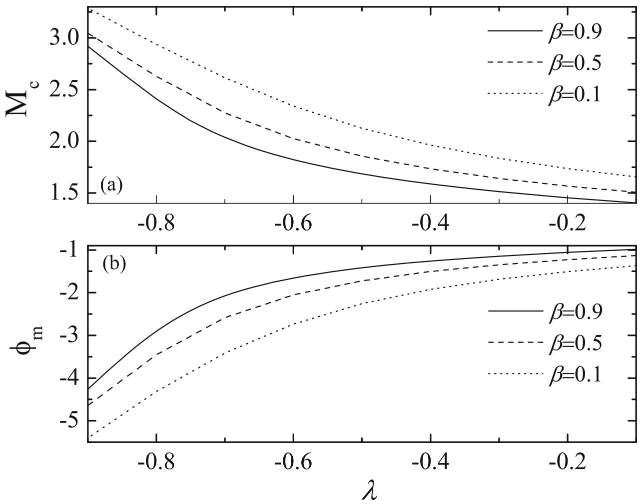

Fig. 1 **a** The dependence of M_c on λ, for different values of β, obtained by Eq. (11). **b** The dependence of the minimum value ϕ_m (corresponding to the maximum amplitude) on λ, for different values of β. The values used are $\beta = 0.1$ (dot line), $\beta = 0.5$ (dash dot line), and $\beta = 0.9$ (solid line); $s = 0.5$

We find from Figs. 1 and 2 that M_c and maximum amplitude decrease when λ becomes larger. The results also reveal that the effect of finite deviation from isothermality of ions changes the Mach number and the amplitude for which localized structures can exist.

It is also found from Fig. 1 that M_c and maximum amplitude decrease with the increase of β. That is to say, when the temperature of free ions keeps away from that of electrons, dust-acoustic solitary structures with larger amplitudes exist and the solitary wave speed is large.

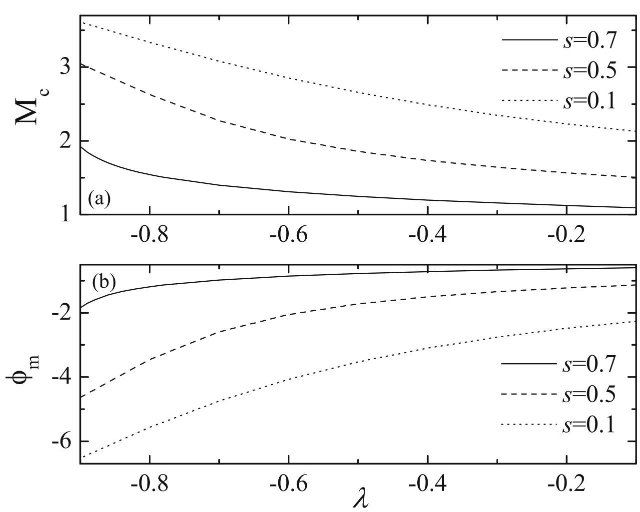

Fig. 2 **a** The dependence of M_c on λ, for different values of s, obtained by Eq. (11). **b** The dependence of the minimum value ϕ_m (corresponding to the maximum amplitude) on λ, for different values of s. The values used are $s = 0.1$ (dot line), $s = 0.5$ (dash dot line), and $s = 0.7$ (solid line), $\beta = 0.5$

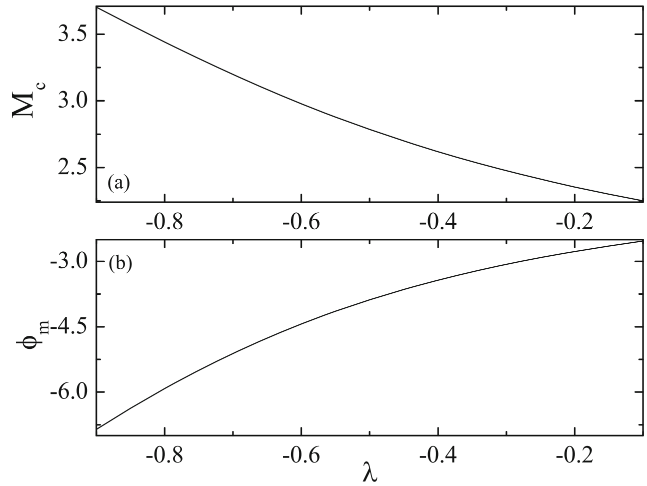

Fig. 3 **a** The dependence of M_c on λ, obtained by Eq. (11). **b** The dependence of the minimum value ϕ_m (corresponding to the maximum amplitude) on λ. $\beta = 0.5$, $s = 0$

It is also found from Fig. 2 that M_c and maximum amplitude decrease with the increasing $s = \frac{n_{e0}}{n_{i0}}$, hence, a complete deletion of the background free electrons owing to the attachment of these electrons to the surface of the dust grains during the charging process can lead to a dust acoustic solitary structure with smaller amplitude. s has larger effect on M_c.

When $s = 0$ ($\nu = 0$), that is, there is no electrons in dusty plasma, the Mach number M_c and amplitude only depend on the trapping parameter λ. We note from Fig. 3 that M_c and amplitude decrease as λ increases.

It is obvious from Eq. (8) that when the amplitude is maximum, the velocity of dust grain u_d equals to, in numerical value, the propagation velocity of solitary wave M_c.

For the special three-dimensional case with the waves only propagating in the z direction, we set that the angle between the direction of wave propagation and magnetic field is 0, which plays an important role in determining the nature of solitary waves. In this case, the Sagdeev potential is identical to that obtained in the one-dimensional case. It can be seen that the magnitude of the external magnetic field has no effect on the amplitude of the solitary waves.

5 Conclusions

In conclusion, we investigate the DAWs for one-dimensional and special three-dimensional cases in dusty plasmas, plasmas with extremely massive, micron-sized, negatively charged inertial dust grains, Boltzmann distibuted electrons and trapped ions. The Sagdeev potential approach has been employed to obtain the dependence of the critical Mach number corresponding to the maximum amplitude of solitary waves on physical quantities. It is observed that the magnitude of the external magnetic field has no effect on the amplitude of the solitary waves. We stress that the results of the present investigation should be useful in understanding the nonlinear features of DAWs in laboratory and space plasmas.

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